Question	Scheme	Marks	AOs
13(a)	Attempts $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin 2t}{\sin t} \quad \left(= 2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \dfrac{2\pi}{3}$ in $\dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin 2t}{\sin t} = \left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $= -\dfrac{1}{dy/dx} = \left(\dfrac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\dfrac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \dfrac{5}{6}, \cancel{-\dfrac{1}{2}}$	M1	2.4
	Substitutes their $\cos t = \dfrac{5}{6}$ into $x = 2\cos t, \ y = \sqrt{3}\cos 2t$,	M1	1.1b
	$Q = \left(\dfrac{5}{3}, \dfrac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
		(13 marks)	

Question 13 continued
Notes:

(a)

M1: Attempts $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$ and achieves a form $k\dfrac{\sin 2t}{\sin t}$ Alternatively candidates may apply the

double angle identity for $\cos 2t$ and achieve a form $k\dfrac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\dfrac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t = \dfrac{2\pi}{3}$ in their $\dfrac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\dfrac{dy}{dx}$. This may be

seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\dfrac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1\Big/\dfrac{dy}{dx}$ with their $\left(-1, -\dfrac{\sqrt{3}}{2}\right)$ to form an equation of the

normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)

M1: For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation
in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form
$y = Ax^2 + B$.

M1: Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$

In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get
an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$

Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $\quad 12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t = \dfrac{5}{6}$ or their $t = \arccos\left(\dfrac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$

If a value of x or y has been found it is for finding the other coordinate.

A1: $Q = \left(\dfrac{5}{3}, \dfrac{7}{18}\sqrt{3}\right)$. Allow $x = \dfrac{5}{3}, y = \dfrac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.