

| Question | Scheme | Marks | AOs |
|------------------|---|------------|------|
| 11. (i) | $x^2 + y^2 + 18x - 2y + 30 = 0 \Rightarrow (x+9)^2 + (y-1)^2 = \dots$ | M1 | 1.1b |
| | Centre $(-9,1)$ | A1 | 1.1b |
| | Gradient of line from $P(-5,7)$ to $(-9,1) = \frac{7-1}{-5+9} = \left(\frac{3}{2}\right)$ | M1 | 1.1b |
| | Equation of tangent is $y-7 = -\frac{2}{3}(x+5)$ | dM1 | 3.1a |
| | $3y-21 = -2x-10 \Rightarrow 2x+3y-11=0$ | A1 | 1.1b |
| | | (5) | |
| (ii) | $x^2 + y^2 - 8x + 12y + k = 0 \Rightarrow (x-4)^2 + (y+6)^2 = 52 - k$ | M1 | 1.1b |
| | Lies in Quadrant 4 if radius $< 4 \Rightarrow "52 - k" < 4^2$ | M1 | 3.1a |
| | $\Rightarrow k > 36$ | A1 | 1.1b |
| | Deduces $52 - k > 0 \Rightarrow$ Full solution $36 < k < 52$ | A1 | 3.2a |
| | | (4) | |
| (9 marks) | | | |

Notes

(i)

M1: Attempts $(x \pm 9)^2 \dots (y \pm 1)^2 = \dots$ It is implied by a centre of $(\pm 9, \pm 1)$

A1: States or uses the centre of C is $(-9, 1)$

M1: A correct attempt to find the gradient of the radius using their $(-9, 1)$ and P . E.g. $\frac{7-1}{-5-(-9)}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7 = -\frac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5 .

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k \in \mathbb{Z}$

.....
Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2 + y^2 + 18x - 2y + 30 = 0 \Rightarrow \dots x + \dots y \frac{dy}{dx} + 18 - 2 \frac{dy}{dx} \dots = 0$

A1: Differentiates $x^2 + y^2 + 18x - 2y + 30 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} + 18 - 2 \frac{dy}{dx} = 0$

M1: Substitutes $P(-5, 7)$ into their equation involving $\frac{dy}{dx}$

.....

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P - k}$

M1: Applying the strategy that it lies entirely within quadrant if “their radius” < 4 and proceeding to obtain an inequality in k only (See scheme). Condone ... ,, 4 for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k$,, 52