18 (a) Use a suitable substitution to show that

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx$$

can be written as

$$\frac{1}{2}\int_a^9 \left(2u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$$

where a is a constant to be found.

[5 marks]

18 (b) Hence, or otherwise, show that

$$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx = \frac{1322}{15}$$

[4 marks]

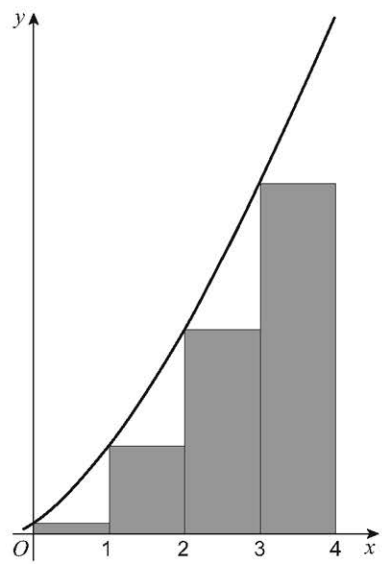
18 (c) A graph has the equation

$$y = (4x+1)\sqrt{2x+1}$$

A student uses four rectangles to approximate the area under the graph between the lines $x = 0$ and $x = 4$

The rectangles are all the same width.

All the rectangles are drawn under the curve as shown in the diagram below.



The total area of the four rectangles is A

The student decides to improve their approximation by increasing the number of rectangles used.

Explain why the value of the student's improved approximation will be greater than A, but less than $\frac{1322}{15}$

[2 marks]